# A Newly Robust Controller Design for the Position Control of Permanent-Magnet Synchronous Motor

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*Abstract—***A robust controller which is designed by employing variable-structure control and linear-quadratic method is presented for a permanent-magnet synchronous motor (PMSM) position control system. It is to achieve accurate control performance in the presence of plant parameter variation and load disturbance. In addition, it possesses the design flexibility of the conventional state feedback control. It is applied to the position control of a PMSM. Simulation and experimental results show that the proposed approach gives a better position response and is robust to parameter variations and load disturbance.**

*Index Terms—***Integral feedback, permanent-magnet synchronous motor, position control, variable-structure control.**

### I. INTRODUCTION

**I** N RECENT YEARS, advancements in magnetic materials, semiconductor power drives, and control theories have made the permanent-magnet synchronous motor (PMSM) drive play a vitally important role in motion-control applications in the low-to-medium-power range. The desired features of the PMSM include its compact structure, high air-gap flux density, high power density, high torque-to-inertia ratio, and high torque capability. When compared with an induction servo motor, a PMSM also has many advantages. For instance, it has the higher efficiency, resulting from the absence of rotor losses and lower no-load current below the rated speed. In addition, its decoupling control performance is far less sensitive to the parameter variations of the motor [1]. To achieve fast four-quadrant operation, smooth starting, and acceleration, the field-oriented control, or vector control, is used in the design of the PMSM drive. Much research has devoted fresh attention to the control of the PMSM [1]–[4], [6].

From the designer's viewpoint, linear state feedback control is theoretically an attractive method for controlling a linear plant represented by a state-space model. The method has the full flexibility of shaping the dynamics of the closed-loop system to

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meet the desired specification. Techniques such as pole placement or linear-quadratic (LQ) method can be used to achieve the designed goals. The motor system usually can be modeled as a second-order state-space system in which the mechanical velocity and position are used as the system states. Thus, these methods, pole placement and LQ method, seem quite suitable to the motor drive system. There are, however, few real motor systems adopting these methods as the controller design. The main problem is that, while the desired performance can be achieved in the nominal system, it is difficult to incorporate robustness consideration into the design procedure.

Considering the optimal control, the LQ method is an easy way to decide the demand control law to satisfy the requirements. It is based on the state-space model. To find the control law, a relative Riccati equation is first solved, and an optimal feedback gain, which will lead to optimal results evaluating from the defined performance index, is obtained. Besides the facts, once the external disturbance and/or the parameters uncertainty exist, then the desired responses may not be obtained.

Thus, if one wants to develop an effective optimal control strategy for the position control of the PMSM drive, one has to overcome the drawbacks mentioned above, including the problems of robustness and keeping the designed flexibility of the pole placement and LQ method.

In the past decade, the variable-structure control (VSC) or sliding-mode control (SMC) strategies have been the focus of many studies and much research, such as in PM synchronous servomotor drive control [2], electrohydraulic position servo control [5], optimal PMSM control [6], and induction motor servo drive control [7], [8]. It is known that the VSC can offer such properties as insensitivity to parameters variations, external disturbance rejection, and fast dynamic response.

Generally, to design a conventional SMC system, there are two design phases that must be considered, namely, the reaching phase and sliding phase. The robustness of a VSC system resides in its sliding phase, but not in its reaching phase. In other words, the closed-loop system dynamic is not completely robust all the time. In addition, while the design technique for the sliding mode has been well established, there is no easy way to shape the dynamics of the reaching phase. Thus, the optimal problem considered in [5] and [6] is weak in the reaching phase and the performance designing.

The proposed optimal control scheme is to meet all the requirements and solve the drawbacks mentioned above. It is designed by combining the LQ method and the VSC method. LQ method is used to decide the demand feedback gain to shape the dynamics and to meet the requirement of the performance index. At the same time, a new VSC strategy is used to conserve the robustness in the optimal control scheme. The system

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Fig. 1. System configuration of field-oriented synchronous motor.

controlled by this VSC strategy will guarantee the robustness for the PMSM position control system.

## II. FIELD-ORIENTED PMSM

The machine model of a PMSM can be described in the rotor rotating reference frame as follows: [1]

$$
v_d = L_d \frac{d}{dt} i_d + R_s i_d - \omega_s L_q i_q \tag{1}
$$

$$
v_q = L_q \frac{d}{dt} i_q + R_s i_q + \omega_s L_d i_d \tag{2}
$$

and

$$
\lambda_q = L_q i_q \tag{3}
$$

$$
\lambda_d = L_d i_d + L_{md} I_{fd}.
$$
\n(4)

In the above equations,  $v_d$  and  $v_q$  are the d, q-axes stator voltages,  $i_d$  and  $i_q$  are the d, q-axes stator currents,  $L_d$  and  $L_q$  are the d, q-axes inductances,  $\lambda_d$  and  $\lambda_q$  are the d, q-axes stator flux linkages, while  $R_s$  and  $\omega_s$  are the stator resistance and inverter frequency, respectively. In (4), the  $I_{fd}$  is the equivalent d-axis magnetizing current, and  $I_{md}$  is the d-axis mutual inductance.

The corresponding electromagnetic torque production is

$$
T_e = \frac{3}{2} p[L_{md} I_{fd} i_q + (L_d - L_q) i_d i_q]
$$
 (5)

The associated electromechanical equations are as follows:

$$
J_m \frac{d\omega_m}{dt} + B_m \omega_m = T_e - T_L \tag{6}
$$

$$
\frac{d\theta_m}{dt} = \omega_m \tag{7}
$$

where  $\omega_m$  is the rotor velocity and  $\theta_m$  is the rotor angular displacement,  $J_m$  is the moment of inertia,  $B_m$  is the damping coefficient, and the inverter frequency is related to the rotor velocity as  $\omega_s = p\omega_m$ .

The primary principle in controlling a PMSM drive is based on field orientation. Since the magnetic flux generated from the PM rotor is fixed in relation to the rotor shaft position, the flux position in the  $d-q$  coordinates can be determined by the shaft position sensor. In (4), if  $i_d = 0$ , the d-axis flux linkage  $\lambda_d$  is fixed. Since  $L_{md}$  and  $I_{fd}$  are constant for a PMSM, the electromagnetic torque  $T_e$  is then proportional to  $i_q$ , which is determined by closed-loop control. The rotor flux is produced only in the  $d$  axis while the current vector is generated in the  $q$  axis in the field-oriented control. Since the generated motor torque is linearly proportional to the  $q$ -axis current, as the  $d$ -axis rotor flux is constant in (4), the maximum torque per ampere can be achieved.

The configuration of a field-oriented PMSM drive system with conventional cascade position and speed control is shown in Fig. 1. The PMSM used in this drive system is a three-phase

where  $p$  is the pole number of the motor.



Fig. 2. Simplified control system block diagram.

four-pole 750-W 3.47-A 3000-r/min type. Using the field-oriented mechanism, the PMSM drive system can be simplified to the control system block diagram shown in Fig. 2, in which

$$
T_e = K_t v \tag{8}
$$

$$
H_p(S) = \frac{1}{J_m s + B_m}.\tag{9}
$$

The parameters of the experimental servo motor system in its nominal condition are  $K_t = 1 \,\mathrm{N\cdot m/V}, J_m = 0.001 \,\mathrm{N\cdot m/s^2},$  and  $B_m = 0.0015 \,\text{N} \cdot \text{m/s}$ , respectively. In addition, v is the inverter torque command which is proportional to the  $q$ -axis current  $i_q$ .

## III. ROBUST CONTROLLER DESIGN BY VARIABLE STRUCTURE

In this section, based on the state-space equation of the motor dynamic system and the introduced performance index, the general concepts of LQ method will be described. Thereafter, under the obtained feedback gain, a new robust controller will be developed to conserve the control performance designed by the LQ method.

# *A. Nominal Condition*

To design a desired controller using the LQ method, the system must first be expressed in the state-space form. Considering the electromechanical equations (6) and (7), they can be expressed in the state-space form (10) with the external load  $T_L$ 

$$
\begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_m}{J_m} \end{bmatrix} \begin{bmatrix} \theta_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_m} \end{bmatrix} T_e - \begin{bmatrix} 0 \\ \frac{1}{J_m} \end{bmatrix} T_L. (10)
$$

For position control, we redefine the new state variables  $x_1$  and  $x_2$  as

$$
\begin{cases}\nx_1 = \theta_m - \theta_d \\
x_2 = \omega_m\n\end{cases} \n\tag{11}
$$

where  $\theta_d$  denotes the position command. Then, combining (10) with (11), the following new state-space equation without considering the disturbance  $T_L$  is obtained:

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}v = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ b \end{bmatrix} v \tag{12}
$$

where  $a = -B_m/J_m$ ,  $b = K_t/J_m$ , and  $\mathbf{x}^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$  is the state.

In view of the LQ method, it is to find an optimal control,  $v^*$ , minimizing the performance index  $J$ 

$$
J = \int_0^\infty \left( \mathbf{x}^T \mathbf{Q} \mathbf{x} + v^T \mathbf{R} v \right) dt \tag{13}
$$

associated with the system (12). In (13), matrix  $\bf{R}$  is positive definite, and  $Q$  is nonnegative definite. To find the optimal control law,  $v^*$ , the following Riccati equation:

$$
\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{b} \mathbf{R}^{-1} \mathbf{b}^T \mathbf{P} + \mathbf{Q} = 0 \tag{14}
$$

is first solved. Let  $\overline{P}$  be the solution for (14) and be nonnegative. Thus, to yield a minimum index of (13), the control law  $v^*$  is as follows:

$$
v^* = -\mathbf{k}^T \mathbf{x} = -\left(\mathbf{R}^{-1} \mathbf{b}^T \overline{\mathbf{P}}\right)^T \mathbf{x}
$$
 (15)

and the feedback gain  $\mathbf{k}^T = [k_1 \quad k_2]$  is defined as

$$
\mathbf{k} = \mathbf{R}^{-1} \mathbf{b}^T \overline{\mathbf{P}}.
$$
 (16)

When system (12) is under the control of (15), the resultant closed-loop dynamics are given by

$$
\dot{\mathbf{x}} = \left[\mathbf{A} - \mathbf{b}\mathbf{k}^T\right]\mathbf{x} = \mathbf{A}_c\mathbf{x}.\tag{17}
$$

# *B. New Switching Surface Design*

Here, a new switching function for sliding-mode position control is designed as follows:

$$
\sigma(\mathbf{x},t) = \mathbf{c}^T[\mathbf{x} - \mathbf{x}_0] - \mathbf{c}^T \mathbf{A}_c \int_0^t \mathbf{x}(\tau) d\tau = 0 \quad (18)
$$

where **x** is system state and  $\mathbf{x}_0$  is its initial value, and  $\mathbf{A}_c$  is defined in (17). In addition,  $\bf{c}$  is a constant vector, which is chosen to make (18) simple but satisfies  $c^T b \neq 0$ . The choice for the current condition is  $\mathbf{c}^T = \begin{bmatrix} 0 & 1/b \end{bmatrix}$ , then  $\mathbf{c}^T \mathbf{b} = 1$ .

It is obvious that, based on (12),  $\sigma(\mathbf{x}, t) = 0$  during all the control process. Therefore, for any chosen state feedback (15), the system possesses a sliding surface  $\sigma(\mathbf{x},t) = 0$  on which the state slides.

# *C. Perturbed Condition*

For most of the system, the perturbation exists, then the linear optimal control  $v^*$  in (15) will neither minimize the performance index (13) nor maintain the sliding mode,  $\sigma(\mathbf{x},t)$  = . Thus, the desired performance will be deteriorated and the steady-state error will occur. To overcome the drawbacks of the LQ method, a VSC-based strategy will be added to the conventional LQ-method-based control system.

For a more realistic condition, the nominal system (12) is rewritten as

$$
\dot{\mathbf{x}} = (\mathbf{A} + \Delta \mathbf{A})\mathbf{x} + (\mathbf{b} + \Delta \mathbf{b})v + \mathbf{d}
$$
  
=  $\begin{bmatrix} 0 & 1 \\ 0 & a + \Delta a \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ b + \Delta b \end{bmatrix} v + \mathbf{d}$  (19)

where  $\triangle A$  and  $\triangle b$  denote the uncertainties introduced by system parameters  $J_m$ ,  $B_m$ , and  $K_t$ , and d represents the external disturbance and its form is as

$$
\mathbf{d} = \begin{bmatrix} 0 \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{T_L}{J_m} \end{bmatrix}.
$$
 (20)

Equation (19) can be expressed in the form of

$$
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}v + \mathbf{p} \tag{21}
$$

where  $\bf{p}$  is the total perturbation given by

$$
\mathbf{p} = \triangle \mathbf{A}\mathbf{x} + \triangle bv + \mathbf{d}.
$$
 (22)

For the position control system, vector  $\bf{p}$  is of the form

$$
\mathbf{p} = \begin{bmatrix} 0 \\ p \end{bmatrix}.
$$

Then, (22) can be rewritten in scalar form as

$$
p = \triangle ax_2 + \triangle bv + d. \tag{23}
$$

One's object is to keep the system states on the sliding surface. Once the sliding mode  $\sigma(\mathbf{x},t) = 0$  can be obtained during all the control process, the control system will reserve as an equivalent system whose dynamics are the same as the closed-loop dynamics in the nominal condition given by (17).

To reserve the nominal responses and control the states on the sliding surface under the perturbed condition, a new control is given as

$$
\overline{v}^* = v^* - q \operatorname{sgn}(\sigma)
$$
  
=  $-\mathbf{k}^T \mathbf{x} - q \operatorname{sgn}(\sigma)$   
=  $-k_1 x_1 - k_2 x_2 - q \operatorname{sgn}(\sigma)$  (24)

where  $sgn(\cdot)$  is a sign function defined as

$$
sgn(\sigma) = \begin{cases} +1, & \text{if } \sigma > 0 \\ -1, & \text{if } \sigma < 0. \end{cases} \tag{25}
$$

 $\sigma$  has been defined in (18) and q is defined as the upper bound of the total perturbation (23), i.e.,

$$
\left|\frac{p}{b}\right| \le q
$$

*Lemma:* If the switching surface  $\sigma(\mathbf{x},t)$  of the controlled system satisfies the following condition, then the existent condition of the sliding mode,  $\sigma(\mathbf{x}, t) = 0$ , must be guaranteed [10]

$$
\sigma \dot{\sigma} < 0. \tag{26}
$$

*Theorem:* The position control given by (24) makes the sliding mode occur and stabilizes the system (19).

*Proof:* According to the lemma, the existence of the sliding mode of the proposed control can be derived as

$$
\sigma\dot{\sigma} = \sigma \left[ \frac{1}{b} \dot{x}_2 + k_1 x_1 + \left( k_2 - \frac{a}{b} x_2 \right) \right]
$$

$$
= \sigma \left\{ \frac{1}{b} [(a + \Delta a)x_2 + (b + \Delta b)v^*] + k_1 x_1 + \left( k_2 - \frac{a}{b} \right) x_2 \right\}.
$$
 (27)

Replacing the control input  $v^*$  with (24), one can obtain the following inequality:

$$
\sigma\dot{\sigma} \le \sigma \left[\frac{p}{b} - q \operatorname{sgn}(\sigma)\right] \le 0. \tag{28}
$$

of the sliding mode in the lemma is satisfied, and the dynamic of the closed-loop system is governed by (17), which is designed stable, so that state  $x(t)$  will slide into the origin.

*Remark:* Because the system is in the sliding mode for the controlled period, the state responses of perturbed system (19) will be totally the same as desired system (17), i.e., it is invariant all the time.

## IV. SIMULATION RESULTS

Simulations are first done using the SIMNON software to verify the proposed control strategy. The controlled object is to drive the motor rotor to rotate  $0.5235$  rad, which is about  $30^{\circ}$ . The parameters of the PMSM drive system in nominal condition have been given in Section II, and substituting them into  $(12)$ , we have the state-space equation as

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1000 \end{bmatrix} v + \begin{bmatrix} 0 \\ 1000 \end{bmatrix} T_L.
$$
 (29)

To determine the feedback gain  $k$  for PMSM drive system, matrices  $Q$  and  $R$  in (13) are chosen as

$$
\mathbf{Q} = \begin{bmatrix} 100 & 0 \\ 0 & 5 \end{bmatrix} \quad \mathbf{R} = 70
$$

and the resultant feedback gain is  $\mathbf{k}^T = [1.1952 \quad 0.2702]$ . The poles of the closed-loop dynamic system are at  $-4.472$  and 267.228, respectively. To reduce the chattering of the SMC, the smooth function is utilized in the control law (24), i.e.,

$$
\overline{v}^*=-\mathbf{k}^T\mathbf{x}-q\frac{\sigma}{|\sigma|+\delta}
$$

where  $\delta$  is a small positive constant chosen as 0.01.

The following approaches are presented for performance comparison.

- 1) *Conventional LQ method:* Let the matrices  $Q$  and  $R$  for the performance index (13) be the same as the choice for the proposed VSC-based approach.
- 2) *LQ method with integral feedback [9]:* To reduce the steady-state error from the external disturbance, the control is fed back through an integrator as is shown in Fig. 3. The performance index is redefined as

$$
J = \int_0^\infty \left( \mathbf{x}^T \mathbf{Q} \mathbf{x} + u^T \mathbf{R} u + \dot{u}^T \mathbf{S} \dot{u} \right) dt
$$

and matrices  $Q$ ,  $R$ , and  $S$  are chosen as

$$
\mathbf{Q} = \begin{bmatrix} 100 & 0 \\ 0 & 5 \end{bmatrix} \quad \mathbf{R} = 0 \quad \mathbf{Q} = 1.
$$

Then, the new system has the system poles at  $-4.472$  and  $33.3712 \pm 33.5039i$ , respectively, and its dominant pole 4.472 is similar to the that of the proposed approach.

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Fig. 3. LQ method with integral feedback.



Fig. 4. Simulated results for nominal system.

Simulated results for the nominal system are presented in Fig. 4. It shows the position responses of three approaches. As one expects, owing to the similar dominant pole of the two LQ-method-based approaches, the trajectories for both systems are similar. As regards the response caused by the proposed VSC-based approach, it also shows the similar response as one expects. It means that the trajectory controlled by the proposed VSC-based optimal approach is totally matched to the nominal system, and this is what one desires. From Fig. 4, it is evident that if one wants the system controlled by the LQ method with integral feedback to have the response similar to that by the LQ method in nominal condition, then some trial-and-error procedures may be needed for the modified LQ method to find a suitable feedback gain to reach this goal, such that the control system with integral feedback can give the similar response as the system without the integral action. Nevertheless, it is not necessary for the proposed new one to reach the goal.

In the following, effects resulting from the external disturbances are given in Fig. 5. In those simulated results, a 1-N m load is suddenly added to the position control system at time 1 s. It is obvious that the position response caused by the LQ method shows a steady-state error, but the steady-state error is zero for the system based on the modified LQ method owing



Fig. 5. Simulated results for system with load added at  $t = 1$  s.



Fig. 6. Simulated results for system with load added at  $t = 0$  s and removed at  $t = 2$  s.

to the integral action. Besides the fact of reducing the steady error to zero, the control system using the modified LQ method is still affected by the disturbance at the instant which the disturbance is added on. Observing the responses controlled by the proposed new one, choosing an appropriate extra control force  $q$ , the system response is independent of the disturbance as Fig. 5 shows. Above all, the response is the same as that of the nominal system before and after the load is added. It is proved that the system controlled by the proposed optimal controller is invariant to the disturbance.

Another consideration for the proposed new optimal controller is that this new one just exists in the sliding phase but not in the reaching phase. i.e., the system controlled by this new one is robust and invariant from the beginning of the control process.



Fig. 7. Pentium-166-based PMSM position control system.

A load with  $1.0$  N $\cdot$ m is added at time 0 s and is removed at time 2 s to evaluate this property. Fig. 6 shows the results under this condition. The system controlled by the LQ method and LQ method with integral feedback is affected at these two instants. However, the system controlled by the proposed method is still little affected; it shows a good property of robustness in the beginning and during all the control process.

### V. EXPERIMENTAL SYSTEM AND RESULTS

## *A. Experimental System Setup*

To practically evaluate the actual performance of the proposed control scheme, a prototype PC-based PMSM optimal position control system is built and tested. The realized system is composed of a Pentium PC, a 750-W PMSM, and another PMSM is axis coupled to the testing motor as the load. The position control algorithms are implemented by a Pentium 166 PC. The position signals are sensed by a 2000 pulse/rev encoder and are fedback to the PC through a 16-bit up/down counter. The corresponding mechanical velocity is computed in the PC. The main program for managing data input and output is written by '86 series assembly language and the proposed optimal control strategy as well as the LQ-method-based control strategy are developed in the mathematical coprocessor language of '387. A sampling frequency of 5 kHz is used in the position and velocity control loop. The data of experimental results are collected in the PC; they are processed and printed out through MATLAB software. The block diagram of this experimental system is shown in Fig. 7.

## *B. Experimental Results*

To show the validity and effectiveness of the proposed control approach, the same position control object as the simulation is adopted, i.e., a 0.5235-rad rotor displacement is set, and the feedback gain for three approaches are all the same as in Section IV; their results will be shown and explained in the following.

Fig. 8 shows the position responses of these three approaches in which the external load is exclusive. In Fig. 8, observing the trace created by the LQ method, due to the motor uncertain parameters, friction and dead band of actuator, etc., the motor



Fig. 8. Experimental position responses without external load.

system controlled by this simple method will result in a steadystate error. As to the trace created by the modified LQ method, owing to the integral action, it exhibits a zero steady-state error, and shows a similar response as compared with the one caused by the proposed new controller. To compare the three trajectories with the simulated results in Fig. 4 for nominal condition, it shows that experimental results match simulation results well except the one caused by the LQ method in Fig. 8, which is affected by the uncertainty and is not considered in the simulation. It is obvious that the desired responses can be easily obtained by the proposed new approach whether or not the uncertainty exists.

Loading effects of these three control approaches are shown in Fig. 9. The same as the setting of the simulations, a 1.0-N·m load is suddenly added to the position system at time 1 s. Fig. 9 shows the position responses controlled by these three approaches. The trace controlled by the LQ method is affected seriously, and the other two traces still reach the desired position. However, the result controlled by the proposed control approach shows a good rejection for external load.



Fig. 9. Experimental position responses with impact load added at  $t = 1$  s.



Fig. 10. Experimental position response with external load added at  $t = 0$  s and removed at  $t = 2$  s.

As regards the problem of the hitting phase for the VSC, Fig. 10 demonstrates these controlled results. In Fig. 10, the traces *(1)* and *(2)* are much affected by the external load at these two instants, which are when the load are added and removed. Above all, for the sake of removing the steady-state error, the modified LQ method has a more serious undershoot and overshoot as compared with the LQ method. However, these effects from the load do not occur for the result controlled by the proposed approach. Particularly, the system is in the sliding phase throughout the control process, as Fig. 11 shows. The control system controlled by the proposed approach completely overcomes the effects resulting from the external disturbance and preserves the desired response as trace *(3)* of Fig. 10 shows.



Fig. 11. Trajectory of switching surface for system of VSC-based controller with the external load added at  $t = 0$  s and removed at  $t = 2$  s.

### VI. CONCLUSIONS

An LQ method with VSC strategy for PMSM position control has been presented. It was shown that the proposed optimal approach is theoretically robust to the plant parameter variations. It can achieve a zero steady-state error for the step input as the LQ method with integral feedback, and it is also invariant to the external load. Simulations and experimental results show that the proposed optimal control strategy can give quite as accurate responses as the nominal condition in the face of external load disturbance and parameter uncertainty.

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